Given here are the solidus and liquidus temperatures for the copper–gold system. Construct the phase diagram for this system and label each region.

<table>
<thead>
<tr>
<th>Composition (wt% Au)</th>
<th>Solidus Temperature (°C)</th>
<th>Liquidus Temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1085</td>
<td>1085</td>
</tr>
<tr>
<td>20</td>
<td>1019</td>
<td>1042</td>
</tr>
<tr>
<td>40</td>
<td>972</td>
<td>996</td>
</tr>
<tr>
<td>60</td>
<td>934</td>
<td>946</td>
</tr>
<tr>
<td>80</td>
<td>911</td>
<td>911</td>
</tr>
<tr>
<td>90</td>
<td>928</td>
<td>942</td>
</tr>
<tr>
<td>95</td>
<td>974</td>
<td>984</td>
</tr>
<tr>
<td>100</td>
<td>1064</td>
<td>1064</td>
</tr>
</tbody>
</table>

Solution

The copper-gold phase diagram is constructed below.
10.5 Cite the phases that are present and the phase compositions for the following alloys:

(a) 25 wt% Pb–75 wt% Mg at 425°C (800°F)
(b) 55 wt% Zn–45 wt% Cu at 600°C (1110°F)
(c) 7.6 lb m Cu and 144.4 lb m Zn at 600°C (1110°F)
(d) 4.2 mol Cu and 1.1 mol Ag at 900°C (1650°F)

Solution

(a) For an alloy composed of 25 wt% Pb–75 wt% Mg and at 425°C, from Figure 10.20, only the α phase is present; its composition is 25 wt% Pb–75 wt% Mg.

(b) For an alloy composed of 55 wt% Zn–45 wt% Cu and at 600°C, from Figure 10.19, β and γ phases are present, and

\[ C_\beta = 51 \text{ wt}\% \text{ Zn-49 wt}\% \text{ Cu} \]
\[ C_\gamma = 58 \text{ wt}\% \text{ Zn-42 wt}\% \text{ Cu} \]

(c) For an alloy composed of 7.6 lb m Cu and 144.4 lb m Zn and at 600°C, we must first determine the Cu and Zn concentrations (using Equation 5.6), as

\[ C_{\text{Cu}} = \frac{7.6 \text{ lb}_m}{7.6 \text{ lb}_m + 144.4 \text{ lb}_m} \times 100 = 5.0 \text{ wt}\% \]
\[ C_{\text{Zn}} = \frac{144.4 \text{ lb}_m}{7.6 \text{ lb}_m + 144.4 \text{ lb}_m} \times 100 = 95.0 \text{ wt}\% \]

From Figure 10.19, only the L phase is present; its composition is 95.0 wt% Zn-5.0 wt% Cu.

(d) For an alloy composed of 4.2 mol Cu and 1.1 mol Ag and at 900°C, it is necessary to determine the Cu and Ag concentrations in weight percent. However, we must first compute the masses of Cu and Ag (in grams) using a rearranged form of Equation 5.7:

\[ m_{\text{Cu}}' = n_{\text{Cu}} A_{\text{Cu}} = (4.2 \text{ mol})(63.55 \text{ g/mol}) = 266.9 \text{ g} \]
\[ m_{\text{Ag}}' = n_{\text{Ag}} A_{\text{Ag}} = (1.1 \text{ mol})(107.87 \text{ g/mol}) = 118.7 \text{ g} \]

Now, using Equation 5.6, concentrations of Cu and Ag are determined as follows:
\[ C_{\text{Cu}} = \frac{266.9 \text{ g}}{266.9 \text{ g} + 118.7 \text{ g}} \times 100 = 69.2 \text{ wt\%} \]

\[ C_{\text{Ag}} = \frac{118.7 \text{ g}}{266.9 \text{ g} + 118.7 \text{ g}} \times 100 = 30.8 \text{ wt\%} \]

From Figure 10.7, \( \alpha \) and liquid phases are present; and

\[ C_{\alpha} = 8 \text{ wt\% Ag-92 w\% Cu} \]
\[ C_L = 45 \text{ wt\% Ag-55 wt\% Cu} \]
Determine the relative amounts (in terms of mass fractions) of the phases for the alloys and temperatures given in Problem 10.5.

Solution

(a) For an alloy composed of 25 wt% Pb-75 wt% Mg and at 425°C, only the α phase is present; therefore $W_\alpha = 1.0$.

(b) For an alloy composed of 55 wt% Zn-45 wt% Cu and at 600°C, compositions of the β and γ phases are

\[ C_\beta = 51 \text{ wt\% Zn-49 wt\% Cu} \]
\[ C_\gamma = 58 \text{ wt\% Zn-42 wt\% Cu} \]

And, since the composition of the alloy, $C_0 = 55 \text{ wt\% Zn-45 wt\% Cu}$, then, using the appropriate lever rule expressions and taking compositions in weight percent zinc

\[ W_\beta = \frac{C_\gamma - C_0}{C_\gamma - C_\beta} = \frac{58 - 55}{58 - 51} = 0.43 \]
\[ W_\gamma = \frac{C_0 - C_\beta}{C_\gamma - C_\beta} = \frac{55 - 51}{58 - 51} = 0.57 \]

(c) For an alloy composed of 7.6 lb_m Cu and 144.4 lb_m Zn (95.0 wt% Zn-5.0 wt% Cu) and at 600°C, only the liquid phase is present; therefore, $W_L = 1.0$.

(d) For an alloy composed of 4.2 mol Cu and 1.1 mol Ag (30.8 wt% Ag-69.2 wt% Cu) and at 900°C, compositions of the α and liquid phases are

\[ C_\alpha = 8 \text{ wt\% Ag-92 w\% Cu} \]
\[ C_L = 45 \text{ wt\% Ag-55 wt\% Cu} \]

And, since the composition of the alloy, $C_0 = 30.8 \text{ wt\% Ag-69.2 wt\% Cu}$, then, using the appropriate lever rule expressions and taking compositions in weight percent silver

\[ W_\alpha = \frac{C_L - C_0}{C_L - C_\alpha} = \frac{45 - 30.8}{45 - 8} = 0.38 \]
\[ W_L = \frac{C_0 - C_\alpha}{C_L - C_\alpha} = \frac{30.8 - 8}{45 - 8} = 0.62 \]
Determine the relative amounts (in terms of volume fractions) of the phases for the alloys and temperatures given in Problems 10.5a and b. Given here are the approximate densities of the various metals at the alloy temperatures:

<table>
<thead>
<tr>
<th>Metal</th>
<th>Temperature (°C)</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cu</td>
<td>600</td>
<td>8.68</td>
</tr>
<tr>
<td>Mg</td>
<td>425</td>
<td>1.68</td>
</tr>
<tr>
<td>Pb</td>
<td>425</td>
<td>10.96</td>
</tr>
<tr>
<td>Zn</td>
<td>600</td>
<td>6.67</td>
</tr>
</tbody>
</table>

**Solution**

This problem asks that we determine the phase volume fractions for the alloys and temperatures in Problems 10.5a and b. This is accomplished by using the technique illustrated in Example Problem 10.3, and also the results of Problems 10.5 and 10.8.

(a) This is a Pb-Mg alloy at 425°C, wherein only the $\alpha$ phase is present. Therefore, $V_\alpha = 1.0$.

(b) This is a Zn-Cu alloy at 600°C, wherein

\[ C_\beta = 51 \text{ wt\% Zn-49 wt\% Cu} \]
\[ C_\gamma = 58 \text{ wt\% Zn-42 wt\% Cu} \]
\[ W_\beta = 0.43 \]
\[ W_\gamma = 0.57 \]
\[ \rho_{Zn} = 6.67 \text{ g/cm}^3 \]
\[ \rho_{Cu} = 8.68 \text{ g/cm}^3 \]

Using these data it is first necessary to compute the densities of the $\beta$ and $\gamma$ phases using Equation 5.13a. Thus

\[ \rho_\beta = \frac{100}{\frac{C_{Zn}(\beta)}{\rho_{Zn}} + \frac{C_{Cu}(\beta)}{\rho_{Cu}}} = \frac{100}{\frac{51}{6.67 \text{ g/cm}^3} + \frac{49}{8.68 \text{ g/cm}^3}} = 7.52 \text{ g/cm}^3 \]
\[
\rho_\gamma = \frac{100}{\frac{C_{\text{Zn}(\gamma)}}{\rho_{\text{Zn}}} + \frac{C_{\text{Cu}(\gamma)}}{\rho_{\text{Cu}}}}
\]
\[
= \frac{100}{\frac{58}{6.67 \text{ g/cm}^3} + \frac{42}{8.68 \text{ g/cm}^3}} = 7.39 \text{ g/cm}^3
\]

Now we may determine the \( V_\beta \) and \( V_\gamma \) values using Equation 10.6. Thus,

\[
V_\beta = \frac{\frac{W_\beta}{\rho_\beta}}{\frac{W_\beta}{\rho_\beta} + \frac{W_\gamma}{\rho_\gamma}}
\]
\[
= \frac{0.43}{\frac{7.52 \text{ g/cm}^3}{0.43} + \frac{7.39 \text{ g/cm}^3}{0.57}} = 0.43
\]

\[
V_\gamma = \frac{\frac{W_\gamma}{\rho_\gamma}}{\frac{W_\beta}{\rho_\beta} + \frac{W_\gamma}{\rho_\gamma}}
\]
\[
= \frac{0.57}{\frac{7.52 \text{ g/cm}^3}{0.43} + \frac{7.39 \text{ g/cm}^3}{0.57}} = 0.57
\]
It is desirable to produce a copper–nickel alloy that has a minimum noncold-worked tensile strength of 380 MPa (55,000 psi) and a ductility of at least 45%EL. Is such an alloy possible? If so, what must be its composition? If this is not possible, then explain why.

Solution

From Figure 10.6a, a tensile strength greater than 380 MPa is possible for compositions between about 32 and 90 wt% Ni. On the other hand, according to Figure 10.6b, ductilities greater than 45%EL exist for compositions less than about 13 wt% and greater than about 94 wt% Ni. Therefore, such an alloy is not possible inasmuch, that in order to meet the stipulated criteria:

\[
\begin{align*}
\text{For } & \text{ a TS } > 380 \text{ MPa} & & 32 \text{ wt} \% < C_{\text{Ni}} < 90 \text{ wt} \% \\
\text{For } & \text{ %EL } > 45\% & & C_{\text{Ni}} < 13 \text{ wt} \% \text{ or } C_{\text{Ni}} > 94 \text{ wt} \% 
\end{align*}
\]
10.16 Briefly explain why, upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases.

Solution

Upon solidification, an alloy of eutectic composition forms a microstructure consisting of alternating layers of the two solid phases because during the solidification atomic diffusion must occur, and with this layered configuration the diffusion path length for the atoms is a minimum.
10.17 Is it possible to have a magnesium–lead alloy in which the mass fractions of primary α and total α are 0.60 and 0.85, respectively, at 460°C (860°F)? Why or why not?

Solution

In order to make this determination we need to set up the appropriate lever rule expression for each of these quantities. From Figure 10.20 and at 460°C, \( C_\alpha = 41 \) wt% Pb, \( C_{Mg_2Pb} = 81 \) wt% Pb, and \( C_{eutectic} = 67 \) wt% Pb.

For primary α

\[
W_{\alpha'} = \frac{C_{eutectic} - C_0}{C_{eutectic} - C_\alpha} = \frac{67 - C_0}{67 - 41} = 0.60
\]

Solving for \( C_0 \) gives \( C_0 = 51.4 \) wt% Pb.

Now the analogous expression for total α

\[
W_\alpha = \frac{C_{Mg_2Pb} - C_0}{C_{Mg_2Pb} - C_\alpha} = \frac{81 - C_0}{81 - 41} = 0.85
\]

which yields a value of 47 wt% Pb for \( C_0 \). Therefore, since these two \( C_0 \) values are different, this alloy is not possible.
10.20 For a 64 wt% Zn–36 wt% Cu alloy, make schematic sketches of the microstructure that would be observed for conditions of very slow cooling at the following temperatures: 900°C (1650°F), 820°C (1510°F), 750°C (1380°F), and 600°C (1100°F). Label all phases and indicate their approximate compositions.

Solution

The illustration below is the Cu-Zn phase diagram (Figure 10.19). A vertical line at a composition of 64 wt% Zn-36 wt% Cu has been drawn, and, in addition, horizontal arrows at the four temperatures called for in the problem statement (i.e., 900°C, 820°C, 750°C, and 600°C).

On the basis of the locations of the four temperature-composition points, schematic sketches of the four respective microstructures along with phase compositions are represented as follows:
Consider 3.5 kg of austenite containing 0.95 wt% C, cooled to below 727°C (1341°F).

(a) What is the proeutectoid phase?
(b) How many kilograms each of total ferrite and cementite form?
(c) How many kilograms each of pearlite and the proeutectoid phase form?
(d) Schematically sketch and label the resulting microstructure.

Solution

(a) The proeutectoid phase will be Fe₃C since 0.95 wt% C is greater than the eutectoid composition (0.76 wt% C).

(b) For this portion of the problem, we are asked to determine how much total ferrite and cementite form. Application of the appropriate lever rule expression yields

\[
W_\alpha = \frac{C_{Fe_3C} - C_0}{C_{Fe_3C} - C_\alpha} \frac{6.70 - 0.95}{6.70 - 0.022} = 0.86
\]

which, when multiplied by the total mass of the alloy, gives (0.86)(3.5 kg) = 3.01 kg of total ferrite.

Similarly, for total cementite,

\[
W_{Fe_3C} = \frac{C_0 - C_\alpha}{C_{Fe_3C} - C_\alpha} \frac{0.95 - 0.022}{6.70 - 0.022} = 0.14
\]

And the mass of total cementite that forms is (0.14)(3.5 kg) = 0.49 kg.

(c) Now we are asked to calculate how much pearlite and the proeutectoid phase (cementite) form. Applying Equation 10.22, in which \(C_1' = 0.95\) wt% C

\[
W_p = \frac{6.70 - C_1'}{6.70 - 0.76} \frac{6.70 - 0.95}{6.70 - 0.76} = 0.97
\]

which corresponds to a mass of (0.97)(3.5 kg) = 3.4 kg. Likewise, from Equation 10.23

\[
W_{Fe_3C'} = \frac{C_1' - 0.76}{5.94} \frac{0.95 - 0.76}{5.94} = 0.03
\]

which is equivalent to (0.03)(3.5 kg) = 0.11 kg of the total 3.5 kg mass.

(d) Schematically, the microstructure would appear as:
Consider 1.5 kg of a 99.7 wt% Fe–0.3 wt% C alloy that is cooled to a temperature just below the eutectoid.

(a) How many kilograms of proeutectoid ferrite form?
(b) How many kilograms of eutectoid ferrite form?
(c) How many kilograms of cementite form?

Solution

(a) Equation 10.21 must be used in computing the amount of proeutectoid ferrite that forms. Thus,

\[ W'_{\alpha} = \frac{0.76 - C'_0}{0.74} = \frac{0.76 - 0.30}{0.74} = 0.622 \]

Or, \((0.622)(1.5 \text{ kg}) = 0.933 \text{ kg}\) of proeutectoid ferrite forms.

(b) In order to determine the amount of eutectoid ferrite, it first becomes necessary to compute the amount of total ferrite using the lever rule applied entirely across the \(\alpha + \text{Fe}_3\text{C}\) phase field, as

\[ W_{\alpha} = \frac{C_{\text{Fe}_3\text{C}} - C'_0}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} = \frac{6.70 - 0.30}{6.70 - 0.022} = 0.958 \]

which corresponds to \((0.958)(1.5 \text{ kg}) = 1.437 \text{ kg}\). Now, the amount of eutectoid ferrite is just the difference between total and proeutectoid ferrites, or

\[ 1.437 \text{ kg} - 0.933 \text{ kg} = 0.504 \text{ kg} \]

(c) With regard to the amount of cementite that forms, again application of the lever rule across the entirety of the \(\alpha + \text{Fe}_3\text{C}\) phase field, leads to

\[ W_{\text{Fe}_3\text{C}} = \frac{C'_0 - C_{\alpha}}{C_{\text{Fe}_3\text{C}} - C_{\alpha}} = \frac{0.30 - 0.022}{6.70 - 0.022} = 0.042 \]

which amounts to \((0.042)(1.5 \text{ kg}) = 0.063 \text{ kg}\) cementite in the alloy.
11.2 (a) For the solidification of nickel, calculate the critical radius \( r^* \) and the activation free energy \( \Delta G^* \) if nucleation is homogeneous. Values for the latent heat of fusion and surface free energy are \(-2.53 \times 10^9 \text{ J/m}^3\) and \(0.255 \text{ J/m}^2\), respectively. Use the supercooling value found in Table 11.1.

(b) Now calculate the number of atoms found in a nucleus of critical size. Assume a lattice parameter of 0.360 nm for solid nickel at its melting temperature.

Solution

(a) This portion of the problem asks that we compute \( r^* \) and \( \Delta G^* \) for the homogeneous nucleation of the solidification of Ni. First of all, Equation 11.6 is used to compute the critical radius. The melting temperature for nickel, found inside the front cover is 1455°C; also values of \( \Delta H_f \) (\(-2.53 \times 10^9 \text{ J/m}^3\)) and \(\gamma\) (0.255 J/m\(^2\)) are given in the problem statement, and the supercooling value found in Table 11.1 is 319°C (or 319 K). Thus, from Equation 11.6 we have

\[
r^* = \left( \frac{2\gamma T_m}{\Delta H_f} \right) \left( \frac{1}{T_m - T} \right)
\]

\[
= \left( \frac{2(0.255 \text{ J/m}^2)(1455 + 273 \text{ K})}{-2.53 \times 10^9 \text{ J/m}^3} \right) \left( \frac{1}{319 \text{ K}} \right)
\]

\[
= 1.09 \times 10^{-9} \text{ m} = 1.09 \text{ nm}
\]

For computation of the activation free energy, Equation 11.7 is employed. Thus

\[
\Delta G^* = \left( \frac{16\pi \gamma^2 T_m^2}{3 \Delta H_f^2} \right) \left( \frac{1}{(T_m - T)^2} \right)
\]

\[
= \left[ \frac{(16\pi)(0.255 \text{ J/m}^2)^3 (1455 + 273 \text{ K})^2}{3(-2.53 \times 10^9 \text{ J/m}^3)^2} \left( \frac{1}{(319 \text{ K})^2} \right) \right]
\]

\[
= 1.27 \times 10^{-18} \text{ J}
\]

(b) In order to compute the number of atoms in a nucleus of critical size (assuming a spherical nucleus of radius \( r^* \)), it is first necessary to determine the number of unit cells, which we then multiply by the number of atoms per unit cell. The number of unit cells found in this critical nucleus is just the ratio of critical nucleus and unit cell volumes. Inasmuch as nickel has the FCC crystal structure, its unit cell volume is just \( a^3 \) where \( a \) is the unit cell length (i.e., the lattice parameter); this value is 0.360 nm, as cited in the problem statement. Therefore, the number of unit cells found in a radius of critical size is just
# unit cells/particle = \frac{4 \pi r^2}{a^3} \cdot (\frac{4}{3})(\pi)(1.09 \text{ nm})^3 \cdot (0.360 \text{ nm})^3 = 116 \text{ unit cells}

Inasmuch as 4 atoms are associated with each FCC unit cell, the total number of atoms per critical nucleus is just
11.7 (a) Briefly describe the phenomena of superheating and supercooling.
    
    (b) Why do these phenomena occur?

Solution

(a) Superheating and supercooling correspond, respectively, to heating or cooling above or below a phase transition temperature without the occurrence of the transformation.

(b) These phenomena occur because right at the phase transition temperature, the driving force is not sufficient to cause the transformation to occur. The driving force is enhanced during superheating or supercooling.
Using the isothermal transformation diagram for an iron–carbon alloy of eutectoid composition (Figure 11.23), specify the nature of the final microstructure (in terms of microconstituents present and approximate percentages of each) of a small specimen that has been subjected to the following time–temperature treatments. In each case assume that the specimen begins at 760°C (1400°F) and that it has been held at this temperature long enough to have achieved a complete and homogeneous austenitic structure.

(a) Rapidly cool to 400°C (750°F), hold for 500 s, then quench to room temperature.
(b) Reheat the specimen in part (a) to 700°C (1290°F) for 20 h.
(c) Cool rapidly to 665°C (1230°F), hold for 10³ s, then quench to room temperature.
(d) Rapidly cool to 350°C (660°F), hold for 150 s, then quench to room temperature.

Solution

This problem asks us to determine the nature of the final microstructure of an iron-carbon alloy of eutectoid composition, that has been subjected to various isothermal heat treatments. Figure 11.23 is used in these determinations.

(a) 100% bainite
(b) 100% spheroidite
(c) 100% coarse pearlite
(d) 50% bainite and 50% martensite
11.13 Make a copy of the isothermal transformation diagram for a 1.13 wt% C iron–carbon alloy (Figure 11.49), and then on this diagram sketch and label time–temperature paths to produce the following microstructures:

(a) 6.2% proeutectoid cementite and 93.8% coarse pearlite
(b) 50% fine pearlite and 50% bainite
(c) 100% martensite
(d) 100% tempered martensite

Solution

Below is shown an isothermal transformation diagram for a 1.13 wt% C iron–carbon alloy, with time–temperature paths that will produce (a) 6.2% proeutectoid cementite and 93.8% coarse pearlite;  (b) 50% fine pearlite and 50% bainite;  (c) 100% martensite;  and (d) 100% tempered martensite.
11.16 Name the microstructural products of 4340 alloy steel specimens that are first completely transformed to austenite, then cooled to room temperature at the following rates:

(a) 0.005°C/s,
(b) 0.05°C/s,
(c) 0.5°C/s,
(d) 5°C/s.

Solution

Figure 11.29 is used for these determinations.

(a) At a cooling rate of 0.005°C/s, proeutectoid ferrite and pearlite form.
(b) At a cooling rate of 0.05°C/s, martensite, ferrite, and bainite form.
(c) At a cooling rate of 0.5°C/s, martensite and bainite form.
(d) At a cooling rate of 5°C/s, martensite and bainite form.
An alloy steel (4340) is to be used in an application requiring a minimum tensile strength of 1515 MPa (220,000 psi) and a minimum ductility of 40% RA. Oil quenching followed by tempering is to be used. Briefly describe the tempering heat treatment.

Solution

We are to consider the tempering of an oil-quenched 4340 steel. From Figure 11.35, for a minimum tensile strength of 1515 MPa (220,000 psi) a tempering temperature of less than 400°C (750°F) is required. Also, for a minimum ductility of 40%AR, tempering must be carried out at a temperature greater than about 300°C (570°F). Therefore, tempering must occur at between 300 and 400°C (570 and 750°F) for 1 h.